



Exponential Waveforms

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Purpose:

Observe the step response of circuits containing dynamic elements.

Equipment Required:

- 1 - Agilent 54622A Deep Memory Oscilloscope or Agilent 54600B Oscilloscope
- 1 - Agilent 33120A Function Generator
- 1 - Protoboard
- 1 - Capacitance Meter (can be shared)
- 1 - 0.1- μF capacitor
- 1 - 1-k Ω resistor
- 1 - 0.47- μF capacitor

Prelab:

Read Sections 8-1 to 8-3 in the text.

Assume the output resistance of the function generator (R_{TH} in Fig. 1) is 50 Ω . Do a CCA run on the circuit of Fig. 1. Use the transient analysis option and an 8 V_{pp} square wave source with no DC offset. You will need to determine an appropriate source frequency so that the voltage across the capacitor reaches its final value before the source changes to the next voltage level. The results of the CCA should look similar to Fig. 2. Print out the CCA results and tape them into your lab journal.

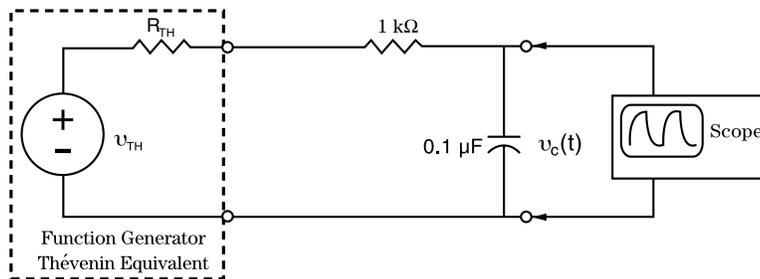


Figure 1

The equation describing the output $v_c(t)$ of the circuit of Fig. 1 is of the form:

$$v_C(t) = (IC - FC) e^{-t/T_c} + FC \tag{1}$$

Assume the function generator produces a square wave with a peak-to-peak voltage of 8 V, no DC offset ($V_{avg} = 0$ V), and a period T_o greater than $10T_c$. Under these circumstances the initial condition IC and the final condition FC will be equal to the maximum and minimum voltages of the source. In this case, the assignment of +4 V or -4 V to the initial condition IC depends on whether the *charging cycle* or the *discharging cycle* are being observed, as shown in Fig. 2. For the discharging cycle the initial condition IC =

+4 V and the final condition $FC = -4$ V. The opposite is true for the charging cycle, $IC = -4$ V and $FC = +4$ V.

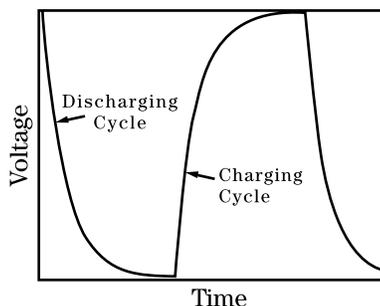


Figure 2

Procedure:

Determine the output resistance of your function generator, R_{TH} . This value is usually printed on a label somewhere on the device. The Thévenin equivalent model of the function generator is shown in Fig. 1. Build the circuit of Fig. 1 on a protoboard.

1. Function generator

Set the function generator to produce a square wave, with a frequency of 50 Hz, no DC offset and a peak-to-peak voltage of 8 V.

2. Oscilloscope

a. Set the oscilloscope to display the waveform $v_c(t)$ produced by this circuit. The display should be similar to Fig. 2. A stable display on the scope is best achieved by triggering the scope with the sync output from the function generator. To do this, connect the sync output from the function generator to the EXT trigger input on the scope. Set the trigger sub-system to use the external signal for triggering.

b. Adjust the scope's horizontal and vertical position knobs and the trigger slope and level so that the waveform is completely visible, but fills the scope face. Line up the start of the exponential waveform with the vertical grid line on the left edge of the scope face. Set the horizontal scale (time base) so the trace looks something like Fig. 3. It is not necessary that you be able to see $5 T_C$; in fact, it is good practice to expand the display so that at least 50% of the scope face is used for measurements (as shown in Fig. 3).

c. Accurately sketch the decaying exponential in your lab journal. The best method is to use a straight edge to draw a full-page-width box that represents the scope face. Count squares on the quadrille to ensure the vertical to horizontal ratio is correct, and that the proper number of grid lines will fit inside the box. Draw in the major grid lines. Then, at each location on the CRT face where the trace intersects a major grid line, draw a dot in the same location on your sketch. Proceed from left to right, until all the intersections are plotted. The final step is to draw a smooth curve through the points. Record the scales and units, along with any other pertinent data, such as the source used to trigger the scope, and the trigger settings. This drawing will be used to measure the circuit time constant T_C .

3. Compute the time constant

Two different methods are suggested for computing the time constant of the circuit of Fig. 1.

a. **Method 1** Compute $0.368(IC - FC)$. The value of $(IC - FC)$ is the peak-to-peak voltage of the source, 8 V. The result, $0.368(8 \text{ V}) = 2.94 \text{ V}$ equates to about 3



divisions up from the bottom of the CRT (Fig. 3). When the exponential has decayed to 36.8% of the difference between the initial voltage and the final voltage, one time constant has elapsed. Measure this time T_c .

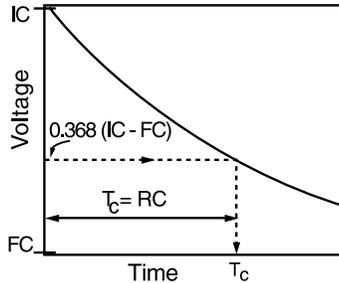


Figure 3

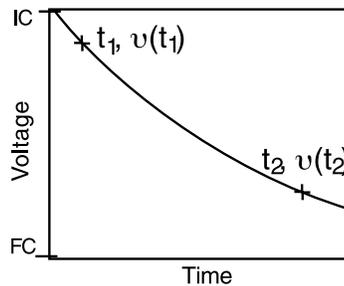


Figure 4

- b. Method 2 Equation 1 can be used to solve for the time constant T_c by measuring the voltage $v_c(t)$ at two different times, as shown in Fig. 4. Moving the last term to the left of the equal sign and dividing these two measurements yields:

$$\frac{v_c(t_1) - FC}{v_c(t_2) - FC} = \frac{(IC - FC)e^{-t_1/T_c}}{(IC - FC)e^{-t_2/T_c}} = e^{(t_2 - t_1)/T_c} \tag{2}$$

Solving for the time constant T_c is accomplished by taking the natural logarithm of both sides and rearranging to produce the result:

$$T_c = \frac{t_2 - t_1}{\ln\left(\frac{v(t_1) - FC}{v(t_2) - FC}\right)} \tag{3}$$

- c. Use the sketch in your journal to compute the time constant T_c using both of these methods.
- d. In the text, we have seen that the time constant is given by the equation $T_c = RC$. Use this equation, either value of T_c computed in Procedure 3c, and the actual value of your resistor plus the output resistance of the function generator to compute the capacitance, C . The result should equal $0.1\mu\text{F} \pm 20\%$. If there is a capacitance meter available in the lab, use it to measure the capacitance. Compute the percent error



between the capacitance measured with the capacitance meter and the capacitance value computed from the time constant T_c .

4. Charging cycle

Set up the scope to display the charging cycle. Sketch the waveform into your journal. Use either of the two methods to determine T_c from this sketch. Are the measured time constants for the charging and the discharging cycles the same?

5. Capacitors in series and parallel

a. Put a $0.47\mu\text{F}$ capacitor in parallel with the $0.1\mu\text{F}$. Measure the new time constant and compute the circuit capacitance from the time constant. (You may omit the scope drawings). Use the measured capacitance to predict an equation to compute the equivalent capacitance for capacitors in parallel.

b. Place the $0.47\mu\text{F}$ and $0.1\mu\text{F}$ capacitors in series. Measure the capacitance using the circuit time constant. What equation could be used to compute the equivalent capacitance of capacitors in series?

Conclusion:

In your conclusion, comment on the accuracy of the methods used to compute the time constant T_c . What are the sources of error? If you measured the value of the capacitor with a capacitance meter, how did your measurements via the time constant compare with the value measured with the capacitance meter?

Roland E. Thomas and Albert J. Rosa, The Analysis and Design of Linear Circuits, Prentice Hall, (New Jersey, 1994)